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Abstract

We study numerically a dynamics of the atom-field coupled system with the assistance of a driving classical field by technique of master equation. It is found, that such a system has nontrivial evolution which essentially depends on the parameters of driving field and in general can reveal the quasi-stochastic behavior. The entanglement coefficient acquires the complicated dynamics and in a dissipative cavity asymptotically tends to a nonzero limit value.

1 Introduction

The ability to create, manipulate, and characterize quantum states is becoming an increasingly important area of physical research, with implications for areas of technology such as quantum computing, quantum cryptography, and communications, see [1], [2], [3], [4] and references therein. Most of research in quantum nonlocality and quantum information is based on entanglement of two-level particles. One of the most interesting aspects of its dynamics is the entanglement between atom and field states. This essentially quantum mechanical property with no classical analog is characterized by the impossibility of completely specifying the state of the global system through the complete knowledge of the individual subsystem's dynamics.

Recently, it was shown that the effective coupling between an atom and a single cavity field mode can be drastically modified in the presence of a strong external driving field [5], [6], [7], [8], [9]. In Refs [10], [11], [12] is proposed a control mechanism of a mesoscopic superposition of states ("Schrödinger cat" state) in a dissipative cavity by the coupling with an external source. The important line of this direction is to use microcavities and microspheres for changing the features of atom-field interaction as a result of placing atom or quantum dots into a microcavity (13], [14],[15] and references therein).

For studying features of atom-field interaction various approximations were proposed: the strong field approximation, dispersive approximation, etc. (see [12], [8], [9] and references therein). Nevertheless the important intermediate field case when the characteristic temporal scales of a quantum system and a classical field are close remains investigated insufficiently. But namely in that regime the disturbance of a coupled atom-field system by a classical field becomes essential. Therefore this case is of significant interest as allows one to control effectively the properties of a quantum system by means of a classical field.

In this Letter we consider the dynamics of atom-field coupled system with assistance of classic driving field beyond of dispersive or strong field approximations. We assume a basic model when a two-level atom interacts with an electromagnetic field in a dissipative cavity. Essential feature of such a process is that at an intermediate driving field a significant amount of coherent states arises. Due to the normalizing condition $Tr\{\rho\}=1$ such coherent states begin to interplay with classical field what gives a rise quite a complicated evolution. Because the complexity of the problem we study such a dynamics by technique of the numerical solution of master equation. This allows us to investigate the behavior of the coupled system in a quite general case. Similar approach was used [16] to study the features of a mesoscopic Josephson junction. Notice that the analysis of density matrix elements provides the important information on the properties of various quantum systems, see [17] and references therein. This paper is organized as follows. In Section 2 we discuss the necessary motion equations: system of the differential equations for density matrix of two-level atom coupled with field cavity modes with assistance of driving field (master equation). Section 3 presents the results of numerical study the evolution of the field and atomic subsystems and the entanglement coefficient. In last Section, we discuss and summarize our results.

2 Basic equations

We consider two-level atom simultaneously interacting with a single-mode cavity field and driving (pumping) by a classical field $\mathbf{E}_c \cos\omega t$, where $|e\rangle$ and $|g\rangle$ being the upper and lower states of an atom, having the transition frequency ω_0 . In the interaction picture the Hamiltonian describing this system, under the standard dipole and rotating wave approximations, is given by[18]

$$\mathbf{V}_{\text{int}} = g\left[e^{-i\delta\tau}\sigma^- a^+ + e^{i\delta\tau}\sigma^+ a\right] + \frac{\Omega}{2}\left[\sigma^x \cos(\Delta\tau) + \sigma^y \sin(\Delta\tau)\right], \tag{1}$$

where $\sigma^{\pm} = \left(\sigma^x \pm i\sigma^y\right)/2$, σ_x , σ_y , σ_z are Pauli matrices. In (1) we have used the next dimensionless variables $\tau = \omega_0 t$, $\delta = 1 - \omega_a/\omega_0$, $\Delta = \omega/\omega_0 - 1$, $g/\omega_0 \to g$, $\Omega/\omega_0 \to \Omega$ (assuming $\hbar = 1$) , a^+ and a are the creation and annihilation operators for the cavity mode, g is the atom-cavity coupling strength, Ω is the Rabi frequency of the classical field, ω_0 is the atomic transition frequency, ω_a is the cavity frequency, and ω is the frequency of the classical field. For the sake of simplicity, here we consider a single two-level atom case. The master equation for a global (field+atom) density operator ρ of the two-level atom coupled with the electromagnetic field in a high-Q cavity is given by

$$\frac{d\rho}{d\tau} = -i[\mathbf{V}_{\text{int}}\rho] + \Im\rho. \tag{2}$$

The losses in the cavity are phenomenologically represented by the term $\Im\rho$. At zero temperature, this part can be written as $\Im\rho = (\gamma/2)\left(2a\rho a^+ - a^+ a\rho - \rho a^+ a\right)$, where γ is the rate of single-photon losses [18]. The Eq.(2) for global density operator ρ governs the evolution of the continuously pumped quantum state. The number states $|n\rangle$ and atomic states $|e\rangle$, $|g\rangle$ are chosen as the basis vectors to solve Eq.(2), in such basis the master equation (2) takes the form

$$\frac{d\rho_{ij}^{nm}}{d\tau} = -igA_{ij}^{nm} - i\frac{\Omega}{2}B_{ij}^{nm} + \frac{\gamma}{2}C_{ij}^{nm}, \tag{3}$$

where

$$A_{ij}^{nm} = \sum_{k=e,g}\left[e^{-i\delta\tau}\sigma_{ik}^-\left(\sqrt{n}\rho_{kj}^{n-1,m} - \sqrt{m+1}\rho_{kj}^{n,m+1}\right) - e^{i\delta\tau}\sigma_{ik}^+\left(\sqrt{m}\rho_{kj}^{n,m-1} - \sqrt{n+1}\rho_{kj}^{n+1,m}\right)\right],$$

$$B_{ee}^{nm} = e^{i\theta}\rho_{eg}^{nm} - e^{-i\theta}\rho_{ge}^{nm},\ B_{gg}^{nm} = e^{-i\theta}\rho_{ge}^{nm} - e^{i\theta}\rho_{eg}^{nm}, B_{eg}^{nm} = e^{i\theta}\left(\rho_{ee}^{nm} - \rho_{gg}^{nm}\right), \theta = \Delta\tau,$$

$$B_{ge}^{nm} = -e^{-i\theta}\left(\rho_{ee}^{nm} - \rho_{gg}^{nm}\right), C_{ij}^{nm} = 2\sqrt{n+1}\sqrt{m+1}\rho_{ij}^{n+1,m+1} - (n+m)\rho_{ij}^{nm},\ \ i,j = e,g.$$

We solve the master equation (3) numerically by the Runge-Kutta method for the ordinary differential equations. The algorithms for integration of such a system numerically can be found, e.g. in Ref.[19]. Such technique allows us to study the details of the dynamics beyond the often used approximations for quantities Δ, Ω , δ and g and in the framework of the unified model. In general $0 \le n, m \le \infty$, but for numerical calculations we have used $0 \le n, m \le M$.

A finite base of the number states M was kept large enough so that the highest energy state is never

populated appreciably.

We suppose the field is initially in the vacuum state $|0\rangle$, whereas the atom is in the upper state $|e\rangle$, so initially $\rho = |0\rangle\langle 0| \otimes |e\rangle\langle e|$. The symbol $\otimes$ represents the tensor product between operators acting on the Hilbert spaces associated with the field and atom. The conditions $Tr\{\rho\} = 1$, $Tr\{\rho^2\} \le 1$ and $\rho = \rho^+$ are checked for the entire evolution. As long the atom-field system does not undergo observation at $\tau \le T$ (T is time of interaction), the evolution of the system is given by the master equation (3).

On the base of numerically calculated global density matrix ρ, we consider the next cases. In the first or second the atom is detected in upper $|e\rangle$ or lower $|g\rangle$ level accordingly. In this cases the detection of the atom projects state ρ into $\rho_e^f = \langle e|\rho|e\rangle / F$ or $\rho_g^f = \langle g|\rho|g\rangle / F$ accordingly, where $F = Tr\{\rho_{e(g)}^f\}$. In third case the atom is not measured, therefore tracing out the operator ρ over the atomic variables, we obtain the reduced field density operator $\rho^f = Tr_a\{\rho\}$. Tracing out the operator ρ over the field variables we calculate the reduced atomic density operator $\rho^a = Tr_f\{\rho\}$. Here $Tr_{a,f}\{\rho\}$ are the partial traces over the atomic or field variables accordingly. The dynamics of these quantities as well the mean numbers is presented in Figs.1-4 while interaction the coupled atom-field system becomes entangled. Such a state cannot be expressed as a product of atom and field contributions. We calculate the evolution of the entanglement coefficient in the last part (Fig.5).

3 Results and discussion

We start calculation from the case which can be associated with a strong driving field approximation (SDFA). In Fig.picig1a the temporal evolution of density matrix elements and the mean photon numbers is shown. We have used the parameters $\Omega = 1$, $g = 0.1$, $\delta = 0$, $\Delta = 0$ and $\gamma = 0$ for $T = 33$, so the condition of strong field $|\Omega| >> |\delta|$, $|g|$ holds. One can see from Fig.1(a), the probability density of photons, initially placed in a vacuum state $\rho_{00}^f(\tau)$, oscillatory decreases from 1 up to zero. The pumping of photons in the upper state ρ_{11}^f becomes, the latter increases to maximum, and, further fades. The off-diagonal parts $\rho_{01}^f(\tau)$,... , $\rho_{04}^f(\tau)$, are excited as well. Since $Tr\{\rho^f(\tau)\} = 1$, it exhibits the dynamic redistribution of the photon states. As a result of action of the driving field the photons leave the vacuum state and transit to higher levels.

One can see from Fig. 1(a) the contribution of the off-diagonal elements, which are responsible for quantum correlations, becomes essential. In Fig. 1(b) the evolution of the mean photon numbers of field states $n(\tau) = Tr(a^+ a\rho^f(\tau))$ and $n_{e,g}(\tau) = Tr(a^+ a\rho_{e,g}^f(\tau))$ obtained from numerical solution Eq.(3) is shown. The comparison of this dynamics with the SDFA [9] provides the understanding of the strong field approximation.

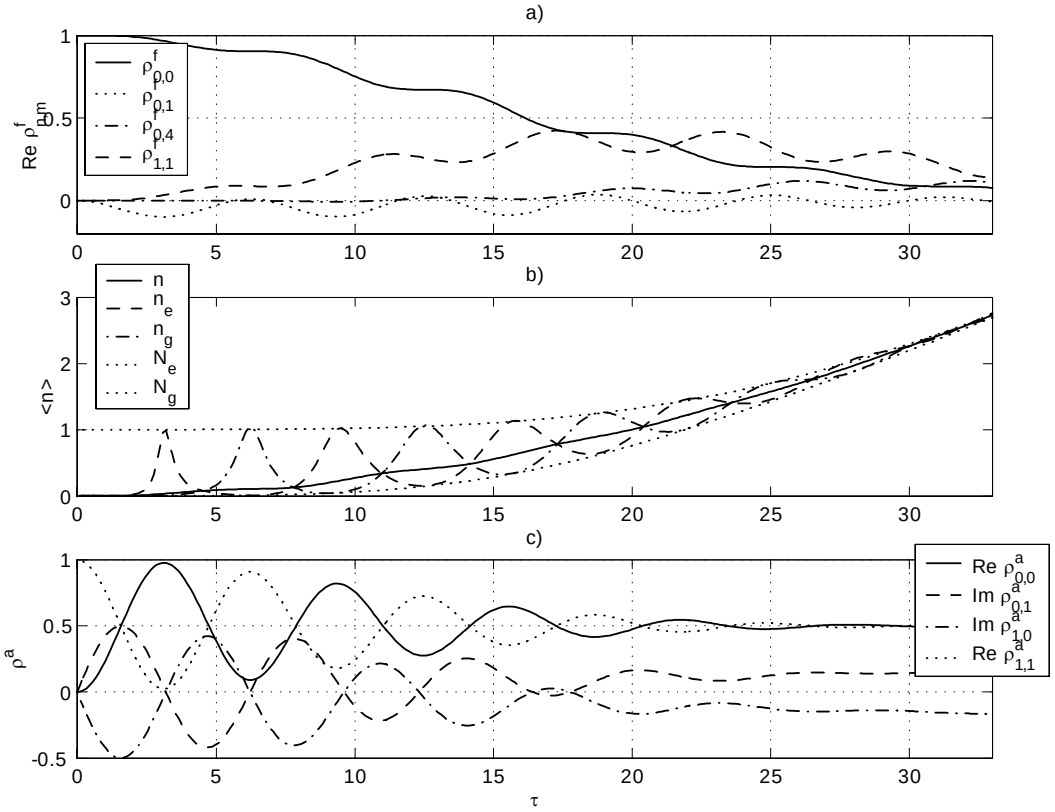

Fig.1. (a) Temporal dynamics of the real part of the matrix elements of the reduced density matrix for T = 33, M = 12, $\gamma = 10^{-4}$, $\delta = \Delta = 0$, $\Omega = 1$ and g = 0.1; (b) mean numbers: n, $n_{e,g}$ for exact numerical solution, $< N >$, $< N_e >$ and $< N_g >$ mean numbers for SDFA (see text for details); (c) the atomic reduced matrix elements ρ^a

To see that clearly we show in Fig.1(b) also the evolution of mean photon numbers $< N >$, $< N_{e,g} >$ derived in SDFA, which are given by (Appendix for details)

$$< N >= N_0(\tau),\ < N_{e,g} >= N_0(\tau)[1 \mp \exp(-2|\xi|^2)]/[1 \pm \exp(-2|\xi|^2)], \qquad (4)$$

where $N_0(\tau) = |\xi|^2$, $\xi = -ig\tau/2$ for $\delta = 0$ case. Corresponding curves are shown in Fig.1(b) by a dotted line. One can see that the data obtained from the exact numerical solution (3), are well-correlated with the strong fields limit case. The mean numbers $< N_{e,g} >$ from (4) are proved to be the envelopes of the Rabi oscillating both mean numbers $n_{e,g}(\tau)$ accordingly. Notice that for the strong field regime the form of such dependences in Fig.1(b) is not sensitive neither to value of $\Omega >> g, \delta$ nor to a base of the number states $M > 12$.

In the same time the lines of $n(\tau)$ and $< N(\tau) >$ practically coincide with each other. Since $n(\tau)$ is proportional to the square of the interaction time T we can conclude that such states are coherent states. From Fig.1(b) is clear that the difference $< N_e(\tau) > - < N_g(\tau) >$ coincides with amplitude of the Rabi oscillations Ω, so in SDFA when $\Omega \to \infty$ the contribution of Rabi oscillations simply can not be discerned.

However in the intermediate driving field case such contribution gives a rise the essential impact to the dynamics of system. So one can see that the numerical solution of the master equation (3) provides more detailed description and allows one to recognize the SDFA as the smoothing of exact dynamics. In Fig.1(c) the behavior of the atomic reduced density matrix $\rho^a(\tau) = Tr_f\{\rho(\tau)\}$ is shown. One can see, that for the used parameters the diagonal matrix elements ρ^a_{00} and ρ^a_{11} asymptotically tend to 0.5 , since the off-diagonal parts are pure imaginary (Hermitian conjugated), that corresponds to a superposition $|\pm\rangle = 2^{-1/2}\left(|e\rangle \pm |g\rangle\right)$ of the atomic states. We can conclude that for strong field regime an atom, initially prepared in upper state, asymptotically tends to $|\pm\rangle$ superposition. In Fig. 2 various the field reduced density matrixes for parameters $\Omega = 1,\ g = 0.1,\ \delta = 0,\ \Delta = 0$ and $\gamma = 0$ at $T = 33$ are shown. Corresponding quantities ρ^f and ρ^a are calculated by the virtue of the procedures tracing and projections discussed above.

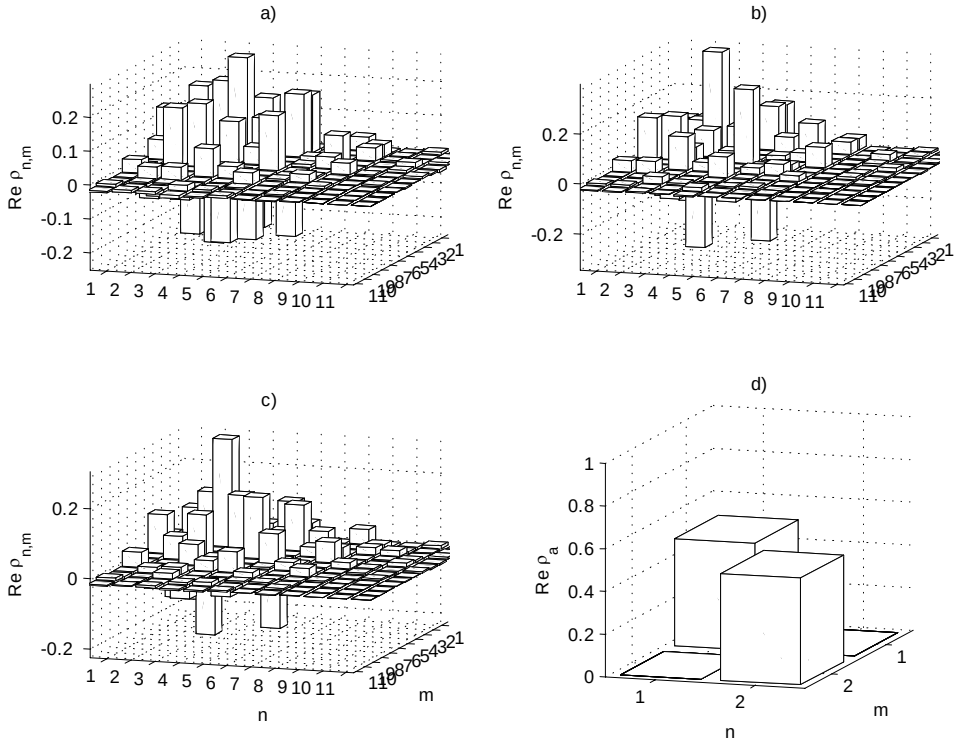

Fig. 2. Real part of the field and atomic reduced density matrixes for T = 33, M = 12, $\gamma = 10^{-4}$, $\delta = \Delta = 0$, Ω, = I and g = 0.1. (a) $\rho_e(\tau)$ (atom is detected in the upper state $|e\rangle$), (b) $\rho_g(\tau)$ (atom is detected in the ground state $|g\rangle$); (c) $\rho^f(\tau)$ (atom is unmeasured); (d) reduced atomic density matrix $\rho^a(\tau)$.

In Fig.2 (a), (b) is shown a real part of the reduced field density matrix $\rho^f_{e(g)}$ for cases, when after time T the atom has been detected in $|e\rangle$ ($|g\rangle$) states accordingly (the imaginary part of ρ^f_e , ρ^f_g and ρ^f vanishes since $\delta = \Delta = 0$).

From Fig.2 (a), (b) one can see that the structure of these states looks quite similar. Indeed our calculations show, that for these states $Tr\{\left(\rho^f_{e,g}(\tau)\right)^2\}=1$, that exhibit a pure state of the field. Though the diagonal elements have prevailing value, the contribution of the off-diagonal elements and the quantum correlations is significant. In Fig.2(c) a real part of the reduced field density matrix $\rho^f = Tr_a\{\rho\}$ is shown. For this case proves to be $Tr\{\left(\rho^f\right)^2\}=0.55$ with two nonzero eigenvalues of ρ^f equal $\lambda_1=0.33$ and $\lambda_2=0.67$. So in unmeasured atom case the field represents a statistical mixture of two superposition states. In Fig. 2(d) the real part of the reduced atomic density matrix ρ^a is shown. From Fig.2 (d) one can conclude that at $T=33$ the atomic state is transformed to the superposition $\left|\pm\right\rangle = 2^{-1/2}\left(\left|e\right\rangle \pm \left|g\right\rangle\right)$, which agrees with Fig.1(c). Now we pass on to situation, when a strong field condition $\Omega >> |g|, |\delta|$ is no longer valid (an intermediate driving field case). Notice that such a case is quite difficult for the analytical studying and is not sufficiently investigated yet.

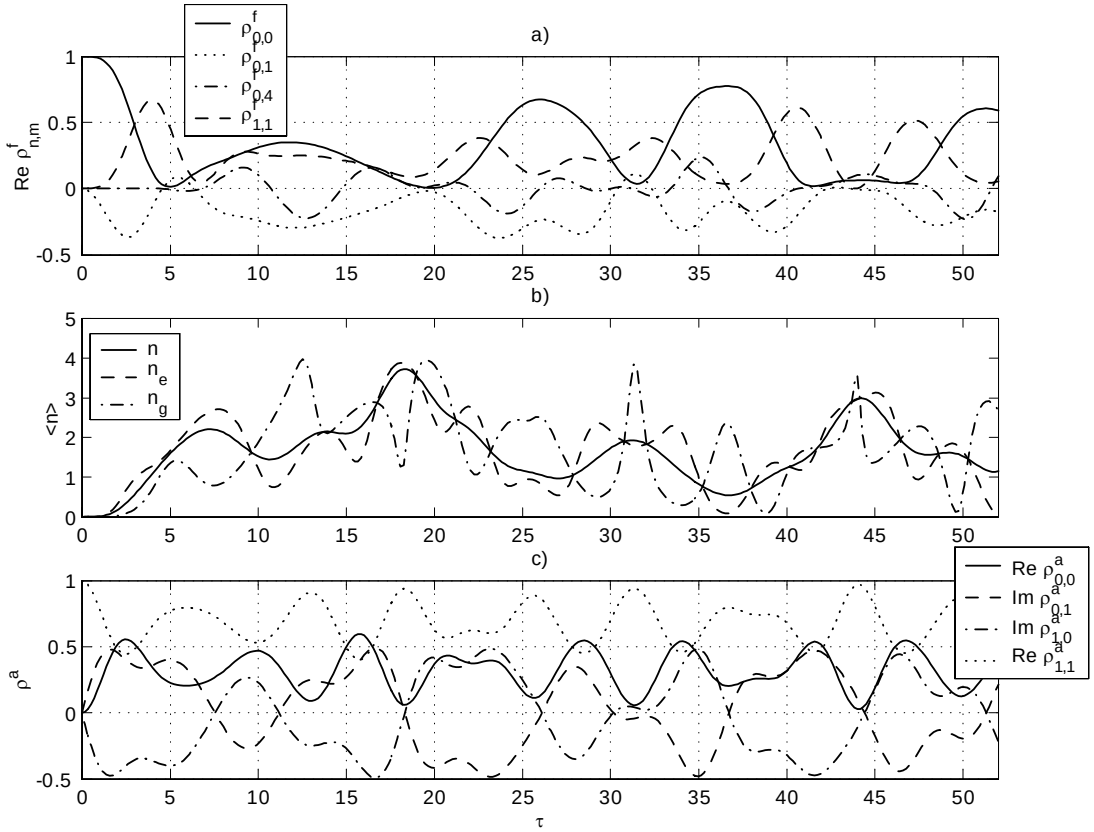

Fig.3. Time dynamics for the same parameters as in Fig.l except g= 0.5. (a) elements of the self reduced density matrix $\rho^f(\tau)$; (b) mean numbers $n(\tau)$, $n_{e,g}(\tau)$; (c) atomic density matrix $\rho^a(\tau)$.

In more details we study the case $\Omega=1$, $g=0.5$ and $\delta=\gamma=\Delta=0$ at the interaction time $T=33$. Corresponding dynamics is shown in Fig.3 and Fig.4. One can see from Fig.3, that now the evolution both matrix elements and the mean numbers look like irregular oscillations.

Fig.3 (a) shows that both ρ_{00}^{f} , and ρ_{11}^{f} undergo the crashes and revivals dynamics. Evolution of the off-diagonal elements and accordingly the dynamics of the quantum correlations become rather irregular and, as one can see from Fig.3 (a), the off-diagonal elements can alternate signs. Fig.3 (b) shows, that for this case the dynamics of the mean numbers essentially differs from the periodicity of a strong field case Fig.1 (a),(b). In this case no appreciable accumulation of energy in the system becomes. Matrix elements ρ^{a} in Fig.3(c) also undergo the complex dynamics.

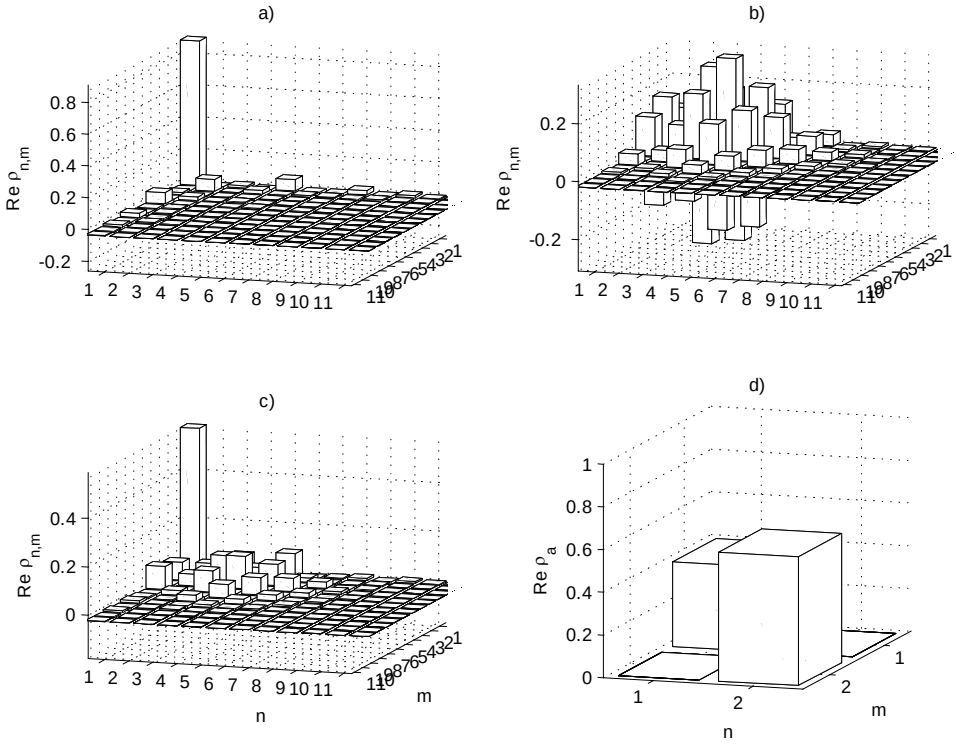

Fig. 4. The structure of the reduced density matrix $\rho_{e,g}^{f}(T)$, $\rho^{f}(T)$ and $\rho^{a}(T)$ at T = 54 for the same parameters as in figure 2 except g = 0.5.

One can see from Fig.4 (a) and Fig.4 (b) that in this case the structure of ρ_{e}^{f} and ρ_{g}^{f} essentially differs. Nevertheless for both cases prove to be $Tr\{\left(\rho_{e,g}^{f}(\tau)\right)^{2}\}=1$, hence these states evolve as pure states. In Fig.4(c) the structure of field density matrix for unmeasured atom ρ^{f} is shown. Our calculation shows that the behavior of $Tr\{\left(\rho^{f}(\tau)\right)^{2}\}$ is rather irregular and normally is the statistical mixture of two field states with $Tr\{\left(\rho^{f}(\tau)\right)^{2}\}<1$. But for some times τ , for instance $\tau=23.3$, the quantity $Tr\{\left(\rho^{f}(\tau)\right)^{2}\}$ assumes the values quite close to 1 , which is signature of a pure field state. One can say that the driving fields can almost decouple a field-atom system for such the temporal intervals. Numerical calculation of the entanglement coefficient dynamics confirms this. Now we discuss the entanglement of the field-atom coupled system with assistance of the driving field.

Notice the pumping action of such field compensates the energy lost to the environment, but not the lost of initial available information about the state (interference of probability amplitudes or coherence) as it is not sensitive to the phase information of the field state [10].

It is well known that decoherence leads to the state evolves into a statistical mixture with $Tr\{\left(\rho^{f}_{e,g}(\tau)\right)^2\} < 1$ and is the main obstacle for construction of the quantum computers. The phenomenon of progressing decoherence was observed experimentally in [4]. Two or more quantum systems are entangled when it is impossible to describe their physical properties by means of a direct product of their respective density operators. The opportunities of numerical experiment allow us to investigate the evolution of the entanglement for the atom-field system in the framework of our model. As a quantitative measure of the entanglement we have used the quantity suggested in Ref. [12]. As it was mentioned we describe the state of field-atom subsystems by the reduced operators of density $\rho^{f,a} = Tr_{a,f}\{\rho\}$. In general, the global state ρ cannot be determined from the reduced states, so for $\tau > 0$ is prove to be $\rho \neq \rho^{f} \otimes \rho^{a}$. The important information on a global state related to both classical and nonlocal (quantum) correlations between the atomic and field subsystems are lost in the partial tracing out procedures. Therefore we can use the difference between a global state ρ and corresponding entanglement noncorrelated state $\rho^{f} \otimes \rho^{a}$ as a quantitative measure of full correlation of a state ρ. Such a difference, which we call as coefficient of the entanglement $\kappa(\tau)$, in general is function of time. We write down the entanglement coefficient as

$$\kappa(\tau) = \left\| \rho - \rho^{f} \otimes \rho^{a} \right\|^2 = Tr\{\left(\rho - \rho^{f} \otimes \rho^{a}\right)^2\}. \qquad (5)$$

To calculate $\kappa(\tau)$ between atom and field, we have substituted in (5) the expressions for the global density matrix ρ and tensor product of reduced operators of field and atom $\rho^{f} \otimes \rho^{a}$, obtained from the numerical solution of master equation (3). Results of calculation are shown in Fig.5.

One can see from Figs.5 (a), that for case of a strong field $\Omega >> g$ and $\delta = 0$ the coefficient $\kappa(\tau)$ increases almost linearly as $\sim n(\tau)^{0.5}$ (solid line). However already in the intermediate field case for $\Omega \sim g$ and $\delta = 0$ such a growth is accompanied by strong irregular oscillations (dash line), that corresponds to the irregular dynamics in Figs.3 (a),(b). One can see from Fig.5 (a) that for some times $\kappa(\tau)$ has minimal values. So for $\tau = 23.3$ we have $\kappa(\tau) \sim 1.5$ and $Tr\{\left(\rho^{f}(\tau)\right)^2\} \sim 0.97$ that answer to an almost pure field state. From the latter a possibility to control the purity of field state by means the amplitude and duration of driving field emerges. In Fig.5 (b) the dynamics of entanglement coefficient $\kappa(\tau)$ is shown for case when loss is not zero. The amplitude of oscillations becomes less if detuning and loss are introduced ($\delta, \Delta, \gamma \neq 0$) (solid line in Fig.5(b)). Further increase γ results in smoothing of oscillations of $\kappa(\tau)$. Numerical solution of the master equation (3) shows, that despite of losses and due to a driving field the system occupies a state with the certain amount of coherent photons for large times $T \to \infty$, at least if γ is small. In this case the entanglement coefficient $\kappa(\tau)$ asymptotically tends $\kappa(\tau) \to \kappa_0 > 0$.

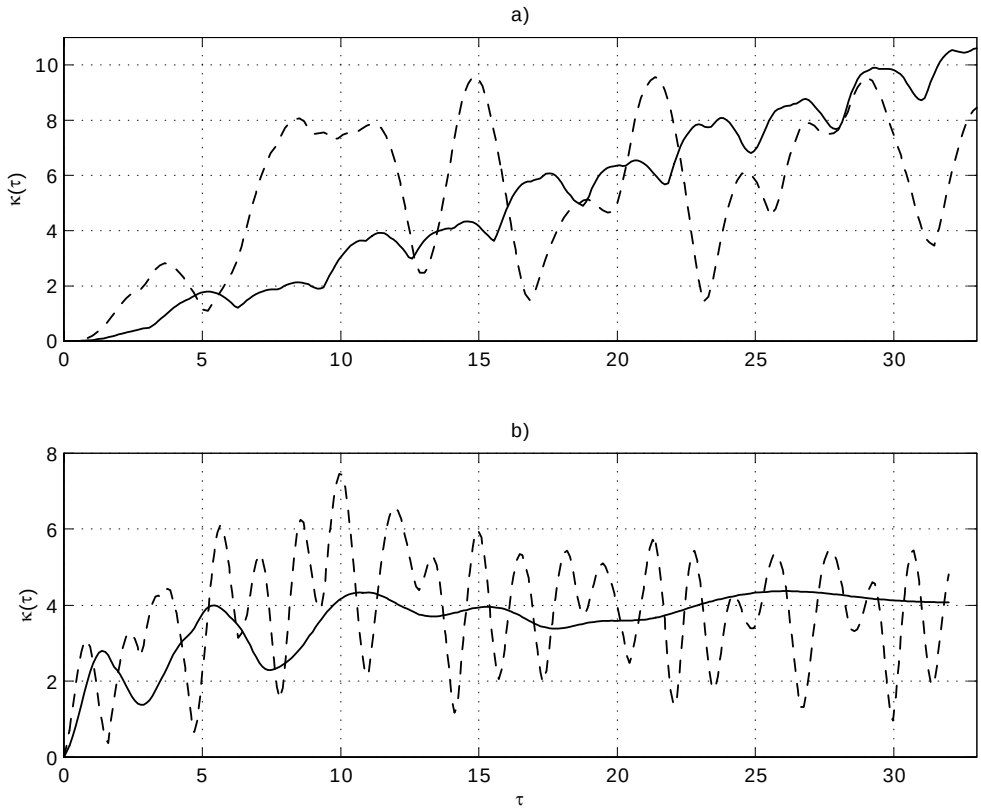

Fig. 5. Temporal dynamics of the entanglement coefficient $\kappa(\tau)$ (see details in text), (a) for solid and dash lines parameters are the same as in Fig.l and Fig.3 accordingly; (b) $\Omega = 1$, $\delta = 0.25$, g = 0.5, $\gamma = 10^{-1}$, $\Delta = 10^{-1}$ (solid line); $\Omega = 1$, $\delta = 10^{-3}$, g = 1, $\gamma = 10^{-2}$, $\Delta = -0.25$ (dash line).

From Fig.5 (b) one can see, that since $\tau = 30$ coefficient $\kappa(\tau)$ tends to the constant value ~ 4 . For the generation of another coherent state it is sufficient to accordingly adjust the field amplitude (Rabi frequency Ω). For weak γ the evolution of $\kappa(\tau)$ has an oscillatory character: for $\gamma << 1$ the field states evolves from a pure state to a mixture and then to a pure state again, see dash line in Fig.5 (b). Thus the driving field prevents the complete disappearance of the field strength, competing with dissipation and by this way it increases the typical decoherence time scales.

4 Conclusion

By numerical solving of the master equation we have shown that in general the atom-field coupled system with assistance of classical driving field has complicate dynamics and evolves in a quantum superposition of coherent states. The quantum coherence effects remain observable, at least if the dissipation is weak enough. In the intermediate field case the system reveals to the quasi-stochastic regime. The entanglement coefficient asymptotically tends to a nonzero limit value. For some temporal intervals the entanglement coefficient becomes minimal, so the driving field can control the entanglement in an atom-field system.

Appendix. Strong driving field approximation (SDFA).

In the fully resonant case $\Delta = 0$ between one mode of a high- Q cavity and a two-level atoms the Hamiltonian (1) acquires more simple form: $\mathbf{V}_{\text{int}} = g\left[e^{-i\delta\tau}\sigma^- a^+ + e^{-i\delta\tau}\sigma^+ a\right] + \frac{\Omega}{2}\sigma^x$. Further simplification is reaching by the unitary transformation $\mathbf{V}^1_{\text{int}} = \exp(A)\mathbf{V}_{\text{int}}\exp(-A)$, where $A = (i\Omega\tau/2)\sigma^x$, which gives

$$\mathbf{V}^1_{\text{int}} = (g/2)\sigma_x\left[e^{-i\delta\tau}a^+ + e^{i\delta\tau}a\right] + i\left(e^{-i\delta\tau}a^+ + e^{i\delta\tau}a\right)\left[\sigma_y\cos\Omega\tau + \sigma_z\sin\Omega\tau\right]. \tag{6}$$

With the assistant of the strong classical driving field, i.e. if $\Omega >> |\delta|$, $|g|$ one can neglect in (6) the terms oscillating fast and then $\mathbf{V}^1_{\text{int}}$ reduces to $\mathbf{V}^1_{\text{int}} = (g/2)\sigma_x\left[e^{-i\delta\tau}a^+ + e^{-i\delta\tau}a\right]$. We consider the unitary evolution operator $U = \exp(\mathbf{V}^1_{\text{int}}\mu(\tau)/\hbar)$, where $\mu(\tau) = \left[\exp(-i\delta\tau) - 1\right]/\delta$, and τ is the interaction time between the atom and the cavity. By applying U to the identity operator $I = |+\rangle\langle+| + |-\rangle\langle-|$, where $|\pm\rangle = 2^{-1/2}\left(|e\rangle \pm |g\rangle\right)$ are the eigenstates of the operator σ_x with eigenvalues ± 1 , we can rewrite U in form

$$U(\xi) = D(\xi)|+\rangle\langle+| + D(-\xi)|-\rangle\langle-| \tag{7}$$

If $\delta = 0$ the parameter $\xi = -ig\tau/2$ is proportional to the interaction time τ. Here $D(\gamma) = \exp(\gamma a^+ - \gamma^* a)$ is well-known unitary displacement operator, where $\underline{\xi}$ is a complex number determining the displacement produced in the associated phase space. Starting from an initial state with the cavity field in a state ρ_{00} and one atom in the $\left(|e\rangle + \alpha|g\rangle\right)/Q$ general state, where $\alpha = const$, $Q = \left(1 + |\alpha|^2\right)^{1/2}$ we obtain $\rho_0 = \rho_{00}\otimes(|e\rangle + \alpha|g\rangle)(\langle e| + \alpha^*\langle g|)$.
Now the density operator evolves to the state $\rho(\xi) = U(\xi)\rho_0 U(-\xi)$ .We consider the cavity is initially in the vacuum state, $\rho_{00} = |0\rangle\langle 0|$.

If one is interested in the cavity field after the interaction without measuring the state of the atom, the cavity field density operator after some algebra can be rewritten in form

$$\rho^f(\xi) = Tr_a\{\rho(\xi)\} = \frac{(\alpha+1)\left(\alpha^*+1\right)}{2\left(1+|\alpha|^2\right)}|\xi\rangle\langle\xi| + \frac{(\alpha-1)\left(\alpha^*-1\right)}{2\left(1+|\alpha|^2\right)}|-\xi\rangle\langle-\xi| \tag{8}$$

which is a statistical mixture of two coherent states with opposite phases.

Other case is if the driven atom is detected after time τ. In the case $\rho_{00} = |0\rangle\langle 0|$, the cavity field density operators after detecting the atom in the upper state $\rho^f_e(\xi)$ or in the lower state $\rho^f_g(\xi)$ are

$$\rho_{e,g}^{f}(\xi)=\frac{1}{2}\frac{\{(1+\alpha)D(\xi)\pm(1-\alpha)D(-\xi)\}\cdot|0\rangle\langle 0|\cdot\{(1+\alpha^{*})D(\xi)\pm(1-\alpha^{*})D(-\xi)\}}{1-|\alpha|^{2}\pm(1+|\alpha|^{2})\exp(-2|\xi|^{2})} \tag{9}$$

In particular if initially the atom was in the upper state $|e\rangle$ we have $\alpha=0$ and $\rho^{f}(\xi)$, $\rho_{e,g}^{f}$ in (8), (9) reduce to [9]

$$\rho^{f}(\xi)=\frac{1}{2}|\xi\rangle\langle\xi|+\frac{1}{2}|-\xi\rangle\langle-\xi| \tag{10}$$

$$\rho_{e,g}^{f}(\xi)=\frac{1}{2R_{\pm}}\{|\xi\rangle\langle\xi|+|-\xi\rangle\langle-\xi|\pm(|\xi\rangle\langle-\xi|+|-\xi\rangle\langle\xi|)\}, \tag{11}$$

where $R_{\pm}=[1\pm\exp(-2\,|\,\xi\,|^{2})]$. Since $Tr\{[\rho_{e,g}^{f}(\xi)]^{2}\}=1$ (11) are pure states of the cavity field. The mean photon number $<N_i>=Tr\{a^{+}a\rho_i(\xi)\}$ of the field states (10) and (11) are

$$<N_f>=N_0, <N_{e,g}>=N_0(R_{\mp}/R_{\pm}), \tag{12}$$

where $N_0=|\xi|^{2}$, $\xi=-ig\tau/2$. If $\delta\neq 0$ one can obtain $\xi=g(\exp(-i\delta\tau)-1)/2\delta$.

5 References

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