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Abstract

We study the problem of well-defined eigenfunctions in parabolic quantum reflectors in two-dimensions. The behaviour of the probability density is analyzed in terms of universal prameters. The standing waves in this geometry are characterized by both continuum and discrete quantum numbers; we discuss symmetry properties and general characteristics.

I. INTRODUCTION

The study of quantum eigen-modes in surfaces bounded by walls made by rows of single adatoms with various geometries is a topic of current interest. Advances in the use of scanning tunneling microscopes [1–3] have allowed the experimental observation of standing-wave patterns on the surface of metallic substrates around impurities and different closed geometries [4].

Quantum mirages within so-called quantum corrals have been the focus of both theoretical and experimental endeavors [5–9]. The electronic dynamics of these systems is characterized as a two-dimensional problem on the surface of the substrate with certain boundary conditions, while the motion in the normal direction is energetically forbidden. Usually, in metallic substrates such as copper, the surface states of the electrons can be described as free quasi-particles with a finite effective mass, which simplifies the theoretical description.

In this work, we have developed the theoretical description of a different system: a quantum reflector consisting of a hard 2D parabolic shaped wall on a substrate as illustrated in Fig. 1. As in a quantum corral, we envision the parabolic reflector to be made by adatoms on the surface, which prevent the motion to the left of the wall. On metallic surfaces, the 2D states can be considered as isotropic with energy dispersion given by

$$E_{\text{surf}} = E_0 + \frac{\hbar^2 k^2}{2m}, \qquad (1)$$

where the energy, E_{surf} is measured with respect to the bottom of the band minimum, E_0, m is the effective mass and k is the two dimensional wave-vector.

II. RESULTS AND DISCUSSION

The problem we are concerned with can be better formulated in parabolic coordinates. The cartesian and parabolic coordinates are related through the equations $x = \frac{1}{2}(\lambda^2 - \mu^2)$ and $y = \lambda\mu$. Thus, the total space is defined by the limits $\lambda \in [0, \infty]$ and $\mu \in [-\infty, \infty]$. The reflector corresponds to the interior problem of the Fig.1 when the boundaries are defined by the equations $\mu = \pm\mu_0$. Here the domain of solutions is defined in the region $\{0 < \lambda < \infty;\ -\mu_0 < \mu < \mu_0\}$. The two-dimensional electron problem in parabolic coordinates is then

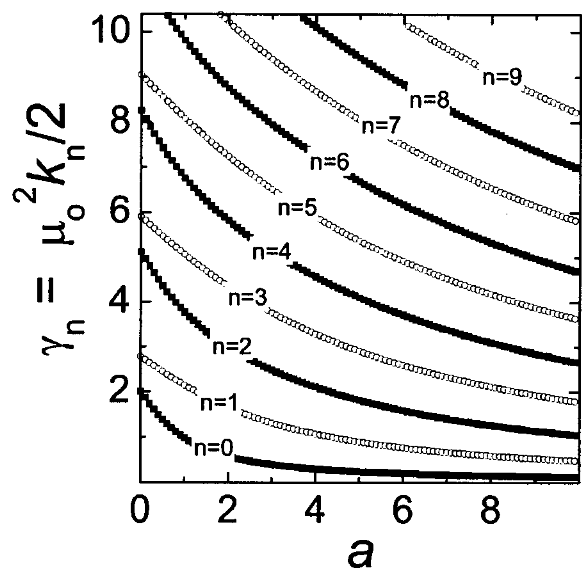

FIG. 2: Energy parameter eigenvalues, γ_n, as function of the parameter a for even and odd states.

and the corresponding F function is

$$F_o\left(a, \lambda^2 k_n\right) = G_o\left(-a, \lambda^2 k_n\right). \tag{7}$$

The eigenvalues of the hard wall problem at $\mu = \mu_0$ (see Fig. 1), are given by the equations

$$G_e\left(a, \mu_0^2 k_n\right) = 0 \quad \text{and} \quad G_o\left(a, \mu_0^2 k_n\right) = 0. \tag{8}$$

These equations provide independent parametric curves for even and odd states, $\gamma_n = \mu_0^2 k_n/2$ as a function of the parameter a as represented in Fig. 2. It is important to note that γ_n is a *universal* value which characterizes the energy of the eigenstate, it is independent of the coefficient μ_0 (or the focal length) of the parabolas, and it is only a function of the parameter a. The dynamics of the wave function is conditioned by the parabolic boundary leaving unbounded the one dimensional movement parallel to the focal axis. This property is responsible for the appearance of continuous energy subbands labeled by the discrete quantum number n. The wave functions for the lower energy states, $n = 0$ and $n = 1$, are represented in Fig. 3. Thus, the 2D parabolic confinement allows for the propagation of quantum standing-waves. Unlike the classical picture of coaxial waves reflecting onto the focus of a parabolic mirror, we may find now in the focus of the parabola a probability density maximum or a minimum (zero) depending on the value of the parameter n. This

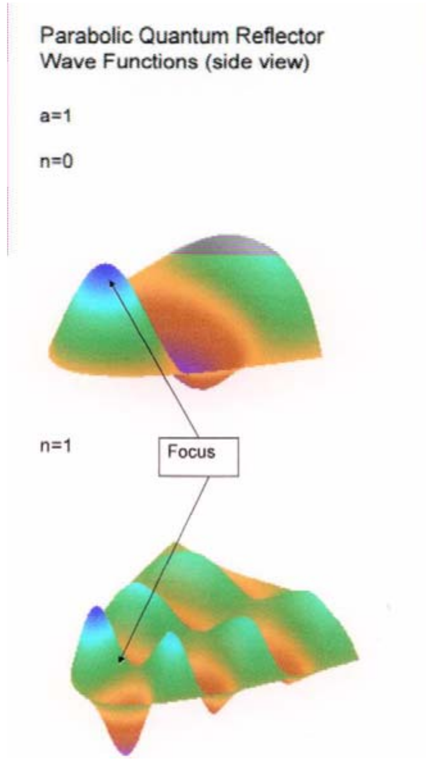

FIG. 3: Calculated wave-functions for $n = 0$ and $n = 1$ with $a = 1$.

would be an important issue concerning the propagation of quantum information contained in the different eigenstates.

The index n characterizes the quantization of the motion perpendicular to the focal axis. Thus the index n determines the parity of the corresponding states with respect to inversion through the axis of the parabola (see the 2D color profile of the wave-functions represented in Fig. 4).

The quantization of the wave functions propagating in the "parabolic wave-guide" has also important effects in the probability density pattern shown in Fig. 4. The value of the quantization index n can be seen to be clearly related to the number of "*valleys*" (nodal lines) that appear along the transverse direction of the parabolas, showing a fan-like pattern on both sides of the focal axis.

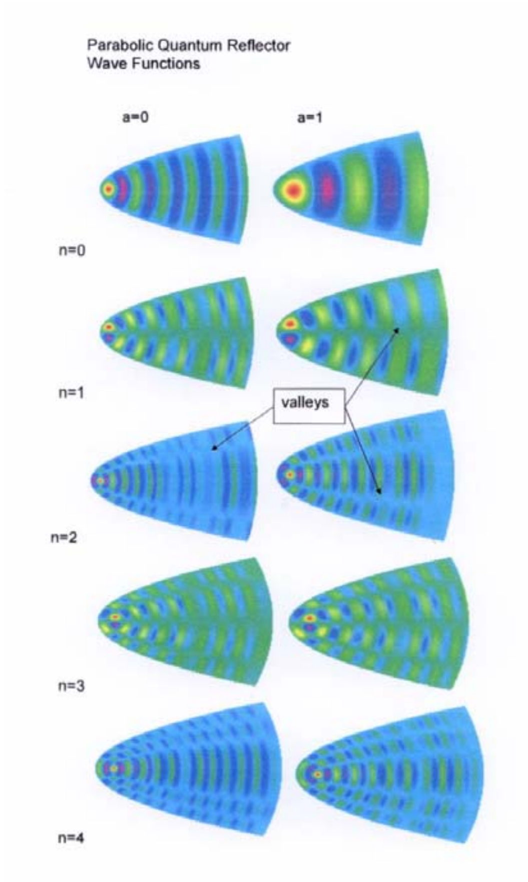

FIG. 4: Wave-functions evolution with the growth of the parameter a ($a = 0$, and $a = 1$) for various values of n.

III. CONCLUSIONS

We have thus characterized the quantization of standing-waves propagating from a parabolic mirror, a "parabolic wave-guide", in two dimensions. The appearance of energy subbands is a quantum effect that is also related to the occurrence of maximum or minimum of the probability density in the focus of the parabola. This effect characterizes the parity of the quantization index n which is also a measure of the number of valleys along the transverse direction to the focal axis.

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