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Abstract

The analysis of necessary conditions for appearance of electromotive force within mobile electroconductive medium in static magnetic field is presented in this paper. The mathematical description of well-known experiments with unipolar induction generator where these effects take place is clarified by applying the Lorentz transform for basic vectors of electromagnetic field in inertial frames of reference. The results of proposed approach have been modeled and tested for estimating their efficiency and accuracy.

1 Introduction

Many various electromechanical devices are widely used in practice and in different branches of science. The physical effects of those devices have a strict mathematical description within the bounds of accepted assumptions. That is why; the well-known results of experiments with electromechanical converters are agreed with the basic statements of corresponding parts of applied mathematics in physics [1], [2]. However, there are some experimental data, which do not have the clear logical and theoretical substantiation yet, particularly, in experiments with unipolar induction generator [3], [4]. The principle of the operations with unipolar generator consists in appearance of electromotive force in static magnetic field during a motion of continuous conductive medium when the results of experiments are invariant to the direction of medium motion (circulating or forward). From a practical point of view the effect of appearance of electromotive force in mobile conductive medium in the magnetic field first of all was investigated by M. Faraday in 1831 [6], [7]. But in spite of an evolution of theoretical electrodynamics some cases of the appearance or absence of electromotive force in unipolar generator still cannot be satisfactorily explained. Sometimes the basic statements of electrodynamics for mobile mediums do not completely agree with well-known results of the practical experiments.
Recently, in scientific literature there are some reports described a behavior of systems with unipolar generator where matching of mathematical substantiation with acquired practical results are looked for [8], [9]. But usually the practical testing of conditions for appearance of electromotive force in unipolar generator requires the development of complicated experimental schemes and devices. Nevertheless, this does not provide the better understanding of nature of the analyzed effects. In order to find adequate explanation of processes taken place in facilities with unipolar generators, novel approaches and descriptive models should be used which widely wrap physical processes in mobile mediums and open the better possibilities for analysis of unknown phenomena. The proposed approach consists in applying Lorentz transform for the vectors of electromagnetic field in inertial frames of reference.

2 Problem formulations

The presence of experimental data, which do not have yet exact theoretical substantiation, prejudices a perfection of the mathematical models used to describe the electromagnetic processes in mobile and static conductive mediums. It reduces significantly possibilities of applying the methods of classical electrodynamics for mathematical modeling of electromagnetic phenomena in electromechanical devices. The generalized results of physical experiments with unipolar generator which can not be explained completely on a theoretical level are presented in the Table 1 [2]. The experiments have been done using the massive electroconductive medium within a static magnetic field. Additionally the measuring circuit which detects the presence of electromotive force in generated loop has been used. The results of experiments have been placed in the different frames of reference providing the measurement of electromotive force as it shown the Table 1.

Table 1 Generalized results of physical experiments

#	Conductive medium	Source of the magnetic field	Measuring circuit	Electromotive force in the measuring circuit
1	mobile	immobile	immobile	present
2	immobile	mobile	immobile	absent
3	immobile	mobile	mobile	present
4	mobile	mobile	immobile	present
5	immobile	immobile	mobile	present
6	mobile	immobile	mobile	absent

2 Mathematical analysis of problem

The scheme with unipolar generator used by Faraday in his research is shown in Fig.1. In these experiments the electroconductive disk 2 is rotated in magnetic field generated by static magnet 1 due to appearance of the electromotive force. The magnitude of electromotive force may be measured as a proportional current or voltage in the electric loop formed by the axis of disc, body of disk, and the measuring circuit, one contact of which slides on the lateral surface of mobile medium (rotated disk). The magnitude of electromotive force is depended on the radius of disk, module of the magnetic induction vector, and angular speed of rotation ω of a mobile medium.

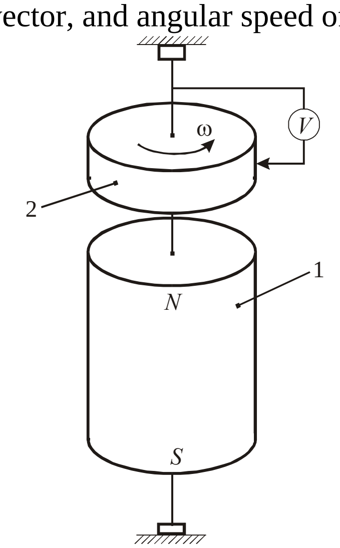

Fig.1. Unipolar induction generator of Faraday

For the analysis of processes in unipolar generator we propose to use Lorentz transform. As known, Lorentz transform is applicable to the inertial frames of reference. But the coordinate system used for description of the analyzed scheme with rotated medium is not inertial. The coordinate system may be accepted as inertial only if the module of linear motion vector for the

most distant points from axis of rotated body is considerably smaller than velocity of light.
The most of circulated electromechanical systems of macrocosm may be described using this condition.
Introducing some notations, the coordinates of immobile system we will define as x, y, z while the coordinates related to a mobile medium are labeled as x', y', z'. Then the vector of electric field strength and the vector of magnetic induction in corresponding reference frames will be defined as E, B and E', B'. Taking into account the conditions of experiment presented in the Fig.1, the vector of electric field strength and the vector of magnetic induction in immobile frame of reference may be describes as

$$\boldsymbol{E}(x,y,z)=\boldsymbol{0}, \qquad \boldsymbol{B}(x,y,z)=\boldsymbol{const}. \tag{1}$$

Correlation between vectors of the electric field strength in different inertial coordinate systems is determined by Lorentz transforms [5]

$$\boldsymbol{E'} = \boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}, \tag{2}$$

where E, E' are the corresponding vectors of electric field strength in the immobile and mobile frames of reference, B is the vector of magnetic induction in the coordinates of $x, y, z,$ and v is the vector of linear velocity of a motion.
In the case of rotary motion for mentioned condition of inertial coordinate system, the equation (2) may be rewritten as

$$\boldsymbol{E'} = \boldsymbol{E} + \boldsymbol{r\omega} \times \boldsymbol{B} \tag{3}$$

where ω is the vector of angular speed of rotation and r is a computed radius.

Taking into account the equations (1), (2), and (3) the electric field strength in coordinates related to a mobile medium is obtained as:

$$\boldsymbol{E'} = \boldsymbol{v} \times \boldsymbol{B}, \tag{4}$$

$$\boldsymbol{E'} = \boldsymbol{r\omega} \times \boldsymbol{B}, \tag{5}$$

Consequently, taking into account the equations (4) and (5) it is possible to note that certain additional electric field strength is presented in the mobile frame of reference comparing it with the equations for x, y, z coordinates. Thus the x, y, z and x', y', z' coordinate systems are not equivalent from the point of view of an energetic relationship. It is because the relations between mechanical systems which belong to different inertial coordinates do not depend on the frame of reference. Therefore, a transition from one coordinate system to another is accompanied by energy discharge or absorption which may be used as quantitative measure of motion relativity. The problem of energy computing of electromagnetic field during a transition from one inertial frame of reference to another remains still open problem and requires additional research [3], [4], [8], [9]. In the case of free motion of electrical charge in a static magnetic field in the immobile system with coordinates of $x, y, z,$ and due to the condition (1) and the equation (4) the additional electric force of electromagnetic nature is appeared and applied to it. Influence of this force on a change changes its motion vector. As a result it has the rotary trajectory within a field of vector B

in *x, y, z* coordinates. Thus, the trajectory of a free electric charge in a space of static magnetic field is a circle or spiral.

There are other physical processes in a continuous electroconductive medium that moves in the magnetic field. During the motion of a body and according to the equation (4) the opposite direction forces will operate on positive and negative electric charges.

Due to the balance of these forces, a conductive medium is not changed, but there is redistribution of electric charges to the value of internal electric strength of field, with direction is opposite to external one. As it known during a motion of dielectric medium its polarization in a static magnetic field is observed. The similar phenomena are detected in experiments with a bullet that carries on itself an electric charge within an electroconductive medium. In a conductive medium also will take place internal redistribution of electric charges, but here the resulting electric field in a medium will be absent. For the computing the electric voltage we will use the well-known formula

$$\boldsymbol{u} = \int_{\boldsymbol{a}}^{\boldsymbol{b}} \boldsymbol{E}\boldsymbol{dl} \tag{6}$$

where u is the voltage between points a and b; E is a resulting vector of electric field strength in space. The equation for finding an electric voltage between arbitrary spatial points in the system of coordinates of $x^{/}$, $y^{/}$, $z^{/}$ according to the condition (1) when the conductive medium is absent is obtained as

$$\boldsymbol{u}' = \int_{\boldsymbol{a}}^{\boldsymbol{b}} \boldsymbol{E}'\boldsymbol{dl} = \int_{\boldsymbol{a}}^{\boldsymbol{b}} \boldsymbol{r}(\omega \times \boldsymbol{B})\boldsymbol{dl}\,. \tag{7}$$

The equation (7) is used only for computing a spatial difference of potentials of an electrostatic field. Because free electric charges are absent here, for example, as it is in experiment with a bullet, the concept of voltage used in analysis of electric circuit is not similar. If there is a motion of electroconductive medium in a static magnetic field the expression (7) with the condition (1) may be rewritten as

$$\boldsymbol{u}' = \int_{\boldsymbol{a}}^{\boldsymbol{b}} \left(\boldsymbol{E}' - \boldsymbol{E}'_{\boldsymbol{in}}\right)\boldsymbol{dl} \tag{8}$$

where $\boldsymbol{E}'_{\boldsymbol{in}}$ is the vector of internal electric field strength in a conductive medium. Because $|\boldsymbol{E}'| = |\boldsymbol{E}'_{\boldsymbol{вн}}|$ the voltage $u^{/} = 0$. Thus, in the frame of reference for mobile conductive medium the electric voltage between arbitrarily selected points will be equal to zero. It corresponds to the result of experiments presented in the row 6 of Table 1. In the coordinate system of *x, y, z* the expression for voltage will be defined as

$$u = -\int_{a}^{b} E'_{вн}\, dl \tag{9}$$

where u is the voltage in the immobile coordinate system. Taking in consideration the voltage computed by the equation (9), the voltage in unipolar induction generator of Faraday may be found by the formula

$$u = -\int_{a}^{b} r(\omega \times B)dl = -\int_{0}^{r} r\omega B dr = -\frac{\omega B R^2}{2} \tag{10}$$

where R is the external radius of electroconductive disk.

Expressions (9) and (10) correspond to the experiments presented in the row 1 of the Table 1. For detailed analysis of the conditions for appearance of electromotive force in unipolar induction generator, it is necessary to examine the processes which take place in the measuring circuit, because it is also a part of formed current loop. The measuring circuit consists of a measuring device and connecting conductors. At the motion of measuring circuit in a static magnetic field the same processes such as in mobile electroconductive medium take place. Taking into account the similar nature of electric charges redistribution along a radius of the disk on an angular coordinate (Fig.1), the physical processes in the conductors of a measuring circuit, and also the method of formation of electric loop the analysis of conditions for appearance of electromotive force in unipolar induction generator may be reduced without the loss of accuracy of approximating model to the analysis of processes in two electrically coupled conductors in rotation within a static magnetic field (Fig.2).

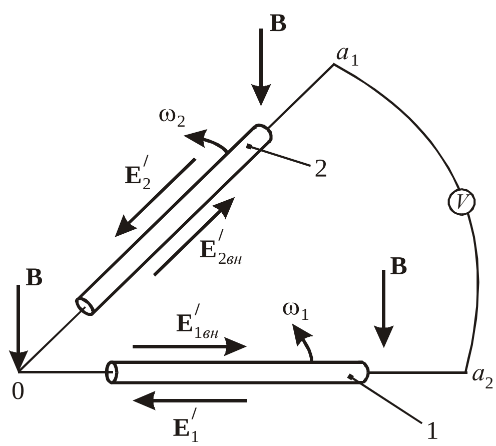

Fig.2. The electric loop, formed by two conductors which are rotated in the static magnetic field

We assume the conductors are rotated in one direction with constant angular speed ω_1 and ω_2. Therefore correlation between the vectors of the electric field strength is defined as

$$E_1' = E_2' ; \quad E'_{1вн} = E'_{2вн} \tag{11}$$

Thus, the voltage in formed electric loop will be equal to:

$$u' = \int_{a2}^{a1} E'_{a1a2} dl = \int_{a2}^{a1} \left(E'_1 - E'_2 - E'_{1вн} + E'_{2вн} \right) dl = 0 \tag{12}$$

The vector of electric field strength $E'_{a1a2} = 0$, it is because in two conductors 1 and 2 in motion to the same direction and with the same speed a similar by magnitude and direction redistribution of electric charges along the conductors take place. In the part of electric loop corresponding to the wires of a measuring circuit (a_1 - a_2), the additional electric field strength described by (5) does not affect the redistribution of electric charges along a coordinate α. Consequently, in an electric loop presented in the Fig.2 the voltage will be equal to zero. It corresponds to the case when a mobile electroconductive medium and a measuring circuit are in the same frame of reference.
The implementation of the discussed electric loop in the induction generator of Faraday implies a mechanical fixing the probe of a measuring circuit on the mobile disk, as a result, the voltage in generator will be equal to zero. We may observe the same results in experiments presented in the rows 2 and 6 of the Table 1.

Considering the case of electric loop with conductors, which are rotated in the same direction but with different angular speeds ω_1 and ω_2 the equation (11) is changed now as:

$$E_1' = E'_{1вн} \neq E_2' = E'_{2вн} ; \tag{13}$$

$$E'_{a1a2} = E'_1 - E'_2 = E'_{2вн} - E'_{1вн} = r\left(\omega_1 \times B - \omega_2 \times B\right) = -r\left(\Delta\omega_{21} \times B\right) \tag{14}$$

The voltage in the measuring circuit according to the equation (12) will be obtained as

$$u = \int_{a2}^{a1} E'_{a1a2} dl = -\int_{a2}^{a1} r\left(\Delta\omega_{21} \times B\right) dl \tag{15}$$

where u is the voltage magnitude in the immobile coordinate system. The equation (15) can be used for computing an electric voltage in frames of reference related to the first or second conductor, but here in the formula (15) the magnitude of module of the magnetic induction vector must correspond to selected coordinate system. If conductors are rotated in the different directions and with different angular speed as it is shown in the Fig. 3, then the voltage in the measuring circuit is found according the following equations.

$$E'_{a1a2} = -E'_1 - E'_2 = E'_{1вн} + E'_{2вн} == -r\left(\omega_1 + \omega_2\right) \times B \tag{16}$$

$$u = \int_{a2}^{a1} E'_{a1a2}\,dl = -\int_{a2}^{a1} r(\omega_1 + \omega_2) \times B\,dl \quad (17)$$

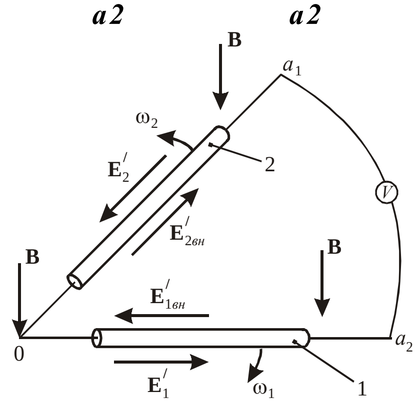

Fig.3. Substitution scheme of electric loop formed by conductors with rotation in different directions

For the cases when $\omega_1 = \omega_2$, the equation (17) is simplified as:

$$u = -\int_{a2}^{a1} 2r(\omega \times B)\,dl \quad (18)$$

We may simplify the analysis of electric loop corresponding to the substitution scheme of unipolar induction generator presented in Fig. 2 for the case when $\omega_1 = 0$. For this condition the substitution scheme may be presented as it shown in Fig.4.

The voltage in measuring circuit for scheme presented in Fig.4 may be obtained according the following equation

$$u = \int_{a2}^{a1} E'_{a1a2}\,dl = -\int_{a2}^{a1} l(\omega \times B)\,dl = -\frac{\omega B l^2}{2} \quad (19)$$

where l is the length of conductor. We obtain the same expression for voltage such as in the case of Faraday's generator presented in the equation (10). Taking into account the expression

$$\omega = \frac{d\alpha}{dt} \quad (20)$$

the equation (19) can be now written as:

$$u = -\frac{d\alpha}{dt}\frac{l^2}{2}B = -\frac{dS}{dt}\frac{d\alpha}{dS}\frac{l^2}{2}B \quad (21)$$

Dependence between the loop element of area dS and the element of angular coordinate $d\alpha$ in the cylindrical frame of reference is obtained as:

$$dS = \frac{l^2}{2} d\alpha \,. \tag{22}$$

Finally, the equation (21) with consideration of formula (22) is rewritten as:

$$u = -\frac{dS}{dt} B \,. \tag{23}$$

Thus, the appearance of electromotive force in the scheme presented in Fig.4 takes place due to changing of the loop area in time and the voltage magnitude does not depend on the absolute value of area in this loop.

The results of experiments presented in the rows 3, 4, and 5 in Table 1 correspond to those obtained by using the scheme shown in the Fig.2 with the condition when $\omega_1 \neq \omega_2$. Due to the equation (15) the voltage is presented in all analyzed cases of electric loops that completely coincide with experimental data. The difference between experiments shown in the rows 3, 4, and 5 consists in the distinct frames of reference related with first or second conductor.

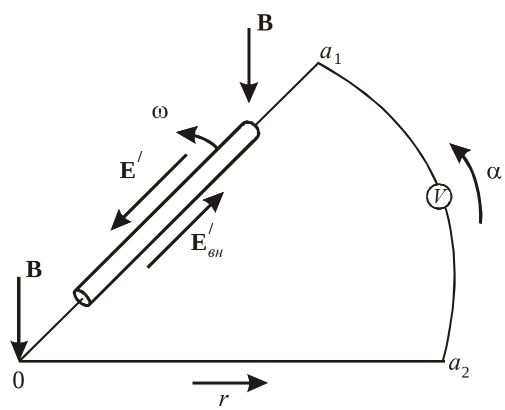

Fig.4. The substitution scheme of unipolar generator of Faraday

4 Conclusions

According to proposed approach the known experimental data with unipolar induction generator now have a mathematical explanation using Lorenz transforms for basic vectors of the electromagnetic field in inertial frames of reference. These results are appropriate and do not contradict to the fundamental rules of electrodynamics of mobile medium.

The principal contribution of this paper is a proposal and experimental confirmation that analysis of appearance of electromotive force in unipolar induction generator must be provided taking into account the physical processes occurred such in electroconductive medium as in a measuring circuit in their own frames of reference.

The results of this paper may be used for construction of mathematical models for different types of electromechanic transformers and also for computing the electric characteristics of unipolar induction generators during their design and construction.

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